APCalculus2021 AB & BC

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

1. The density of a bacteria population in a circular petri dish at a distance r centimeters from the center of the dish is given by an increasing, differentiable function f, where $f(r)$ is measured in milligrams per square centimeter. Values of $f(r)$ for selected values of r are given in the table above.

(a) Use the data in the table to estimate $f'(2.25)$. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) The total mass, in milligrams, of bacteria in the petri dish is given by the integral expression $2\pi\int_0^4 r f(r)\,dr$. Approximate the value of $2\pi\int_0^4 r f(r)\,dr$ using a right Riemann sum with the four subintervals indicated by the data in the table.

(c) Is the approximation found in part (b) an overestimate or underestimate of the total mass of bacteria in the petri dish? Explain your reasoning.

(d) The density of bacteria in the petri dish, for $1 \le r \le 4$, is modeled by the function g defined by $g(r) = 2 - 16(\cos(1.57\sqrt{r}))^3$. For what value of k, $1 < k < 4$, is $g(k)$ equal to the average value of $g(r)$ on the interval $1 \le r \le 4$?

(a)

$f'(2.25) \approx \dfrac{f(2.5)-f(2)}{2.5-2} = \dfrac{10-6}{0.5} = 8$	Estimate	**1 point**
At a distance of $r = 2.25$ centimeters from the center of the petri dish, the density of the bacteria population is increasing at a rate of 8 milligrams per square centimeter per centimeter.	Interpretation with units	**1 point**

(b)

$2\pi\int_0^4 r f(r)\,dr \approx 2\pi(1\cdot f(1)\cdot(1-0) + 2\cdot f(2)\cdot(2-1) + 2.5\cdot f(2.5)\cdot(2.5-2) + 4\cdot f(4)\cdot(4-2.5))$	Right Riemann sum setup	**1 point**
$= 2\pi(1\cdot 2\cdot 1 + 2\cdot 6\cdot 1 + 2.5\cdot 10\cdot 0.5 + 4\cdot 18\cdot 1.5)$ $= 269\pi = 845.088$	Approximation	**1 point**

(c)

$\frac{d}{dr}(r\,f(r)) = f(r) + r\,f'(r)$	Product rule expression for $\frac{d}{dr}(r\,f(r))$	**1 point**
Because f is nonnegative and increasing, $\frac{d}{dr}(r\,f(r)) > 0$ on the interval $0 \le r \le 4$. Thus, the integrand $r\,f(r)$ is strictly increasing. Therefore, the right Riemann sum approximation of $2\pi\int_0^4 r\,f(r)\,dr$ is an overestimate.	Answer with explanation	**1 point**

(d)

Average value $= g_{avg} = \frac{1}{4-1}\int_1^4 g(r)\,dr$	Definite integral	**1 point**
$\frac{1}{4-1}\int_1^4 g(r)\,dr = 9.875795$	Average value	**1 point**
$g(k) = g_{avg} \Rightarrow k = 2.497$	Answer	**1 point**

2. A particle, P, is moving along the x-axis. The velocity of particle P at time t is given by $v_P(t) = \sin\left(t^{1.5}\right)$ for $0 \le t \le \pi$. At time $t = 0$, particle P is at position $x = 5$.

   A second particle, Q, also moves along the x-axis. The velocity of particle Q at time t is given by $v_Q(t) = (t - 1.8) \cdot 1.25^t$ for $0 \le t \le \pi$. At time $t = 0$, particle Q is at position $x = 10$.

   (a) Find the positions of particles P and Q at time $t = 1$.

   (b) Are particles P and Q moving toward each other or away from each other at time $t = 1$? Explain your reasoning.

   (c) Find the acceleration of particle Q at time $t = 1$. Is the speed of particle Q increasing or decreasing at time $t = 1$? Explain your reasoning.

   (d) Find the total distance traveled by particle P over the time interval $0 \le t \le \pi$.

(a)

$x_P(1) = 5 + \int_0^1 v_P(t)\,dt = 5.370660$ At time $t = 1$, the position of particle P is $x = 5.371$ (or 5.370). $x_Q(1) = 10 + \int_0^1 v_Q(t)\,dt = 8.564355$ At time $t = 1$, the position of particle Q is $x = 8.564$.	One definite integral	**1 point**
	One position	**1 point**
	The other position	**1 point**

(b)

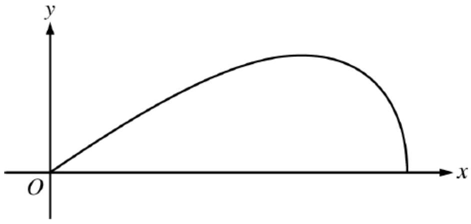
$v_P(1) = \sin(1^{1.5}) = 0.841471 > 0$ At time $t = 1$, particle P is moving to the right.	Direction of motion for one particle	**1 point**
$v_Q(1) = (1 - 1.8) \cdot 1.25^1 = -1 < 0$ At time $t = 1$, particle Q is moving to the left. At time $t = 1$, $x_P(1) < x_Q(1)$, so particle P is to the left of particle Q. Thus, at time $t = 1$, particles P and Q are moving toward each other.	Answer with explanation	**1 point**

(c)

$a_Q(1) = v_Q'(1) = 1.026856$ The acceleration of particle Q is 1.027 (or 1.026) at time $t = 1$.	Setup and acceleration	**1 point**
$v_Q(1) = -1 < 0$ and $a_Q(1) > 0$ The speed of particle Q is decreasing at time $t = 1$ because the velocity and acceleration have opposite signs.	Speed decreasing with reason	**1 point**

(d)

$\int_0^{\pi} \lvert v_P(t) \rvert \, dt = 1.93148$ Over the time interval $0 \le t \le \pi$, the total distance traveled by particle P is 1.931.	Definite integral	**1 point**
	Answer	**1 point**



3. A company designs spinning toys using the family of functions $y = cx\sqrt{4 - x^2}$, where c is a positive constant. The figure above shows the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$, for some c. Each spinning toy is in the shape of the solid generated when such a region is revolved about the x-axis. Both x and y are measured in inches.

(a) Find the area of the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$ for $c = 6$.

(b) It is known that, for $y = cx\sqrt{4 - x^2}$, $\frac{dy}{dx} = \frac{c\left(4 - 2x^2\right)}{\sqrt{4 - x^2}}$. For a particular spinning toy, the radius of the largest cross-sectional circular slice is 1.2 inches. What is the value of c for this spinning toy?

(c) For another spinning toy, the volume is 2π cubic inches. What is the value of c for this spinning toy?

(a)

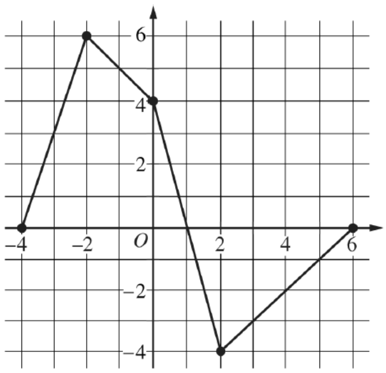
$6x\sqrt{4 - x^2} = 0 \Rightarrow x = 0,\ x = 2$ Area $= \int_0^2 6x\sqrt{4 - x^2}\,dx$	Integrand	**1 point**
Let $u = 4 - x^2$. $du = -2x\,dx \Rightarrow -\frac{1}{2}\,du = x\,dx$ $x = 0 \Rightarrow u = 4 - 0^2 = 4$ $x = 2 \Rightarrow u = 4 - 2^2 = 0$ $\int_0^2 6x\sqrt{4 - x^2}\,dx = \int_4^0 6\left(-\frac{1}{2}\right)\sqrt{u}\,du = -3\int_4^0 u^{1/2}\,du = 3\int_0^4 u^{1/2}\,du$ $= 2u^{3/2}\Big\vert_{u=0}^{u=4} = 2 \cdot 8 = 16$	Antiderivative	**1 point**
The area of the region is 16 square inches.	Answer	**1 point**

(b)

The cross-sectional circular slice with the largest radius occurs where $cx\sqrt{4-x^2}$ has its maximum on the interval $0 < x < 2$. $\frac{dy}{dx} = \frac{c(4-2x^2)}{\sqrt{4-x^2}} = 0 \Rightarrow x = \sqrt{2}$	Sets $\frac{dy}{dx} = 0$	**1 point**
$x = \sqrt{2} \Rightarrow y = c\sqrt{2}\sqrt{4-(\sqrt{2})^2} = 2c$ $2c = 1.2 \Rightarrow c = 0.6$	Answer	**1 point**

(c)

Volume $= \int_0^2 \pi\left(cx\sqrt{4-x^2}\right)^2 dx = \pi c^2 \int_0^2 x^2(4-x^2)\, dx$	Form of the integrand Limits and constant	**1 point** **1 point**
$= \pi c^2 \int_0^2 (4x^2 - x^4)\, dx = \pi c^2 \left(\frac{4}{3}x^3 - \frac{1}{5}x^5 \Big\|_0^2\right)$	Antiderivative	**1 point**
$= \pi c^2\left(\frac{32}{3} - \frac{32}{5}\right) = \frac{64\pi c^2}{15}$ $\frac{64\pi c^2}{15} = 2\pi \Rightarrow c^2 = \frac{15}{32} \Rightarrow c = \sqrt{\frac{15}{32}}$	Answer	**1 point**



Graph of f

4. Let f be a continuous function defined on the closed interval $-4 \le x \le 6$. The graph of f, consisting of four line segments, is shown above. Let G be the function defined by $G(x) = \int_0^x f(t)\, dt$.

(a) On what open intervals is the graph of G concave up? Give a reason for your answer.

(b) Let P be the function defined by $P(x) = G(x) \cdot f(x)$. Find $P'(3)$.

(c) Find $\lim_{x \to 2} \frac{G(x)}{x^2 - 2x}$.

(d) Find the average rate of change of G on the interval $[-4, 2]$. Does the Mean Value Theorem guarantee a value c, $-4 < c < 2$, for which $G'(c)$ is equal to this average rate of change? Justify your answer.

(a)

$G'(x) = f(x)$ The graph of G is concave up for $-4 < x < -2$ and $2 < x < 6$, because $G' = f$ is increasing on these intervals.	Answer with reason	**1 point**

(b)

$P'(x) = G(x) \cdot f'(x) + f(x) \cdot G'(x)$ $P'(3) = G(3) \cdot f'(3) + f(3) \cdot G'(3)$	Product rule	**1 point**
Substituting $G(3) = \int_0^3 f(t)\,dt = -3.5$ and $G'(3) = f(3) = -3$ into the above expression for $P'(3)$ gives the following:	$G(3)$ or $G'(3)$	**1 point**
$P'(3) = -3.5 \cdot 1 + (-3) \cdot (-3) = 5.5$	Answer	**1 point**

(c)

Using L'Hospital's Rule, $\lim_{x \to 2} \frac{G(x)}{x^2 - 2x} = \lim_{x \to 2} \frac{G'(x)}{2x - 2}$ $= \lim_{x \to 2} \frac{f(x)}{2x - 2} = \frac{f(2)}{2} = \frac{-4}{2} = -2$	Answer with justification	**1 point**

(d)

$G(2) = \int_0^2 f(t)\,dt = 0$ and $G(-4) = \int_0^{-4} f(t)\,dt = -16$ Average rate of change $= \frac{G(2) - G(-4)}{2 - (-4)} = \frac{0 - (-16)}{6} = \frac{8}{3}$	Average rate of change	**1 point**
Yes, $G'(x) = f(x)$ so G is differentiable on $(-4, 2)$ and continuous on $[-4, 2]$. Therefore, the Mean Value Theorem applies and guarantees a value c, $-4 < c < 2$, such that $G'(c) = \frac{8}{3}$.	Answer with justification	**1 point**

5. Consider the function $y = f(x)$ whose curve is given by the equation $2y^2 - 6 = y \sin x$ for $y > 0$.

(a) Show that $\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$.

(b) Write an equation for the line tangent to the curve at the point $(0, \sqrt{3})$.

(c) For $0 \le x \le \pi$ and $y > 0$, find the coordinates of the point where the line tangent to the curve is horizontal.

(d) Determine whether f has a relative minimum, a relative maximum, or neither at the point found in part (c). Justify your answer.

(a)

$\frac{d}{dx}\left(2y^2 - 6\right) = \frac{d}{dx}(y \sin x) \Rightarrow 4y\frac{dy}{dx} = \frac{dy}{dx}\sin x + y\cos x$	Implicit differentiation	**1 point**
$\Rightarrow 4y\frac{dy}{dx} - \frac{dy}{dx}\sin x = y\cos x \Rightarrow \frac{dy}{dx}(4y - \sin x) = y\cos x$ $\Rightarrow \frac{dy}{dx} = \frac{y\cos x}{4y - \sin x}$	Verification	**1 point**

(b)

At the point $(0, \sqrt{3})$, $\frac{dy}{dx} = \frac{\sqrt{3}\cos 0}{4\sqrt{3} - \sin 0} = \frac{1}{4}$. An equation for the tangent line is $y = \sqrt{3} + \frac{1}{4}x$.	Answer	**1 point**

(c)

$\frac{dy}{dx} = \frac{y\cos x}{4y - \sin x} = 0 \Rightarrow y\cos x = 0$ and $4y - \sin x \neq 0$	Sets $\frac{dy}{dx} = 0$	**1 point**
$y\cos x = 0$ and $y > 0 \Rightarrow x = \frac{\pi}{2}$	$x = \frac{\pi}{2}$	**1 point**
When $x = \frac{\pi}{2}$, $y\sin x = 2y^2 - 6 \Rightarrow y\sin\frac{\pi}{2} = 2y^2 - 6$ $\Rightarrow y = 2y^2 - 6 \Rightarrow 2y^2 - y - 6 = 0$ $\Rightarrow (2y + 3)(y - 2) = 0 \Rightarrow y = 2$ When $x = \frac{\pi}{2}$ and $y = 2$, $4y - \sin x = 8 - 1 \neq 0$. Therefore, the line tangent to the curve is horizontal at the point $\left(\frac{\pi}{2}, 2\right)$.	$y = 2$	**1 point**

(d)

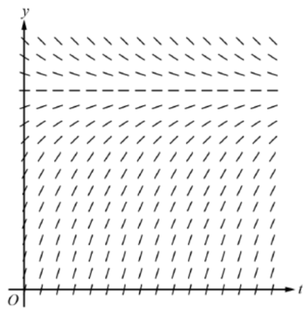
$\frac{d^2y}{dx^2} = \frac{(4y - \sin x)\left(\frac{dy}{dx}\cos x - y\sin x\right) - (y\cos x)\left(4\frac{dy}{dx} - \cos x\right)}{(4y - \sin x)^2}$	Considers $\frac{d^2y}{dx^2}$	**1 point**
When $x = \frac{\pi}{2}$ and $y = 2$, $\frac{d^2y}{dx^2} = \frac{\left(4\cdot 2 - \sin\frac{\pi}{2}\right)\left(0\cdot\cos\frac{\pi}{2} - 2\cdot\sin\frac{\pi}{2}\right) - \left(2\cos\frac{\pi}{2}\right)\left(4\cdot 0 - \cos\frac{\pi}{2}\right)}{\left(4\cdot 2 - \sin\frac{\pi}{2}\right)^2}$ $= \frac{(7)(-2) - (0)(0)}{(7)^2} = \frac{-2}{7} < 0.$	$\frac{d^2y}{dx^2}$ at $\left(\frac{\pi}{2}, 2\right)$	**1 point**
f has a relative maximum at the point $\left(\frac{\pi}{2}, 2\right)$ because $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$.	Answer with justification	**1 point**

(d’)

For the function $y = f(x)$ near the point $\left(\frac{\pi}{2}, 2\right)$, $4y - \sin x > 0$ and $y > 0$.	Considers sign of $4y - \sin x$	**1 point**
Thus, $\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$ changes from positive to negative at $x = \frac{\pi}{2}$.	$\frac{dy}{dx}$ changes from positive to negative at $x = \frac{\pi}{2}$	**1 point**
By the First Derivative Test, f has a relative maximum at the point $\left(\frac{\pi}{2}, 2\right)$.	Conclusion	**1 point**

6. A medication is administered to a patient. The amount, in milligrams, of the medication in the patient at time t hours is modeled by a function $y = A(t)$ that satisfies the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$. At time $t = 0$ hours, there are 0 milligrams of the medication in the patient.

(a) A portion of the slope field for the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ is given below. Sketch the solution curve through the point $(0, 0)$.

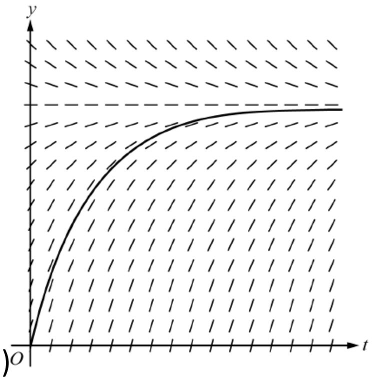


(b) Using correct units, interpret the statement $\lim_{t \to \infty} A(t) = 12$ in the context of this problem.

(c) Use separation of variables to find $y = A(t)$, the particular solution to the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ with initial condition $A(0) = 0$.

(d) A different procedure is used to administer the medication to a second patient. The amount, in milligrams, of the medication in the second patient at time t hours is modeled by a function $y = B(t)$ that satisfies the differential equation $\frac{dy}{dt} = 3 - \frac{y}{t + 2}$. At time $t = 1$ hour, there are 2.5 milligrams of the medication in the second patient. Is the rate of change of the amount of medication in the second patient increasing or decreasing at time $t = 1$? Give a reason for your answer.

(a)



	Solution curve	**1 point**

(b)

Over time the amount of medication in the patient approaches 12 milligrams.	Interpretation	**1 point**

(c)

$\frac{dy}{dt} = \frac{12 - y}{3} \Rightarrow \frac{dy}{12 - y} = \frac{dt}{3}$	Separation of variables	**1 point**
$\int \frac{dy}{12 - y} = \int \frac{dt}{3} \Rightarrow -\ln\lvert 12 - y\rvert = \frac{t}{3} + C$	Antiderivatives	**1 point**
$\ln\lvert 12 - y\rvert = -\frac{t}{3} - C \Rightarrow \lvert 12 - y\rvert = e^{-t/3-C}$ $\Rightarrow y = 12 + Ke^{-t/3}$ $0 = 12 + K \Rightarrow K = -12$	Constant of integration and uses initial condition	**1 point**
$y = A(t) = 12 - 12e^{-t/3}$	Solves for y	**1 point**

(d)

$\frac{dy}{dt} = 3 - \frac{y}{t + 2} \Rightarrow \frac{d^2y}{dt^2} = (-1)\frac{\frac{dy}{dt}(t + 2) - y}{(t + 2)^2}$	Quotient rule	**1 point**
$B'(1) = 3 - \frac{B(1)}{3} = 3 - \frac{2.5}{3} = \frac{6.5}{3}$ $B''(1) = -\frac{B'(1) \cdot 3 - B(1)}{3^2} = -\frac{6.5 - 2.5}{9} = -\frac{4}{9} < 0$	$B''(1) < 0$	**1 point**
The rate of change of the amount of medication is decreasing at time $t = 1$ because $B''(1) < 0$ and $\frac{d^2y}{dt^2}$ is continuous in an interval containing $t = 1$.	Answer with reason	**1 point**

BC

2. For time $t \ge 0$, a particle moves in the xy-plane with position $(x(t), y(t))$ and velocity vector $\left\langle (t-1)e^{t^2}, \sin\left(t^{1.25}\right) \right\rangle$. At time $t = 0$, the position of the particle is $(-2, 5)$.

(a) Find the speed of the particle at time $t = 1.2$. Find the acceleration vector of the particle at time $t = 1.2$.

(b) Find the total distance traveled by the particle over the time interval $0 \le t \le 1.2$.

(c) Find the coordinates of the point at which the particle is farthest to the left for $t \ge 0$. Explain why there is no point at which the particle is farthest to the right for $t \ge 0$.

(a)

$\sqrt{(x'(1.2))^2 + (y'(1.2))^2} = 1.271488$ At time $t = 1.2$, the speed of the particle is 1.271.	Speed	**1 point**
$\langle x''(1.2), y''(1.2) \rangle = \langle 6.246630, 0.405125 \rangle$ At time $t = 1.2$, the acceleration vector of the particle is $\langle 6.247 \text{ (or } 6.246), 0.405 \rangle$.	Acceleration vector	**1 point**

(b)

$\int_0^{1.2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 1.009817$	Integrand	**1 point**
The total distance traveled by the particle over the time interval $0 \le t \le 1.2$ is 1.010 (or 1.009).	Answer	**1 point**

(c)

$x'(t) = (t-1)e^{t^2} = 0 \Rightarrow t = 1$	Sets $x'(t) = 0$	**1 point**
Because $x'(t) < 0$ for $0 < t < 1$ and $x'(t) > 0$ for $t > 1$, the particle is farthest to the left at time $t = 1$.	Explains leftmost position at $t = 1$	**1 point**
$x(1) = -2 + \int_0^1 x'(t)\, dt = -2.603511$ $y(1) = 5 + \int_0^1 y'(t)\, dt = 5.410486$	One coordinate of leftmost position	**1 point**
The particle is farthest to the left at point $(-2.604 \text{ (or } -2.603), 5.410)$.	Leftmost position	**1 point**
Because $x'(t) > 0$ for $t > 1$, the particle moves to the right for $t > 1$. Also, $x(2) = -2 + \int_0^2 x'(t)\, dt > -2 = x(0)$, so the particle's motion extends to the right of its initial position after time $t = 1$. Therefore, there is no point at which the particle is farthest to the right.	Explanation	**1 point**

5. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = y \cdot (x \ln x)$ with initial condition $f(1) = 4$. It can be shown that $f''(1) = 4$.

(a) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(2)$.

(b) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(2)$. Show the work that leads to your answer.

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = y \cdot (x \ln x)$ with initial condition $f(1) = 4$.

(a)

$f'(1) = \frac{dy}{dx}\Big\vert_{(x,\,y)=(1,\,4)} = 4 \cdot (1 \ln 1) = 0$ The second-degree Taylor polynomial for f about $x = 1$ is $f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 = 4 + 0(x-1) + \frac{4}{2}(x-1)^2$.	Polynomial	**1 point**
$f(2) \approx 4 + 2(2-1)^2 = 6$	Approximation	**1 point**

(b)

$f(1.5) \approx f(1) + 0.5 \cdot \frac{dy}{dx}\Big\vert_{(x,\,y)=(1,\,4)} = 4 + 0.5 \cdot 0 = 4$ $f(2) \approx f(1.5) + 0.5 \cdot \frac{dy}{dx}\Big\vert_{(x,\,y)=(1.5,\,4)}$	Euler's method with two steps	**1 point**
$\approx 4 + 0.5 \cdot 4 \cdot (1.5 \ln 1.5) = 4 + 3 \ln 1.5$	Answer	**1 point**

(c)

$\frac{1}{y}\,dy = x \ln x\,dx$	Separation of variables	**1 point**
Using integration by parts, $u = \ln x \quad du = \frac{1}{x}\,dx$ $dv = x\,dx \quad v = \frac{x^2}{2}$ $\int x \ln x\,dx = \frac{x^2}{2} \cdot \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x}\,dx = \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C$	Antiderivative for $x \ln x$	**1 point**
$\ln\lvert y\rvert = \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C$	Antiderivative for $\frac{1}{y}$	**1 point**
$\ln 4 = 0 - \frac{1}{4} + C \Rightarrow C = \ln 4 + \frac{1}{4}$	Constant of integration and uses initial condition	**1 point**
$y = e^{\left(\frac{x^2 \ln x}{2} - \frac{x^2}{4} + \ln 4 + \frac{1}{4}\right)}$ Note: This solution is valid for $x > 0$.	Solves for y	**1 point**

6. The function g has derivatives of all orders for all real numbers. The Maclaurin series for g is given by

$$g(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2e^n + 3}$$ on its interval of convergence.

(a) State the conditions necessary to use the integral test to determine convergence of the series $\sum_{n=0}^{\infty} \frac{1}{e^n}$.

Use the integral test to show that $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges.

(b) Use the limit comparison test with the series $\sum_{n=0}^{\infty} \frac{1}{e^n}$ to show that the series $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ converges absolutely.

(c) Determine the radius of convergence of the Maclaurin series for g.

(d) The first two terms of the series $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ are used to approximate $g(1)$. Use the alternating series error bound to determine an upper bound on the error of the approximation.

(a)

e^{-x} is positive, decreasing, and continuous on the interval $[0, \infty)$.	Conditions	**1 point**
To use the integral test to show that $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges, show that $\int_0^{\infty} e^{-x}\,dx$ is finite (converges).	Improper integral	**1 point**
$\int_0^{\infty} e^{-x}\,dx = \lim_{b\to\infty} \int_0^{b} e^{-x}\,dx = \lim_{b\to\infty} \left(-e^{-x}\Big\|_0^b\right) = \lim_{b\to\infty} \left(-e^{-b} + e^0\right) = 1$ Because the integral $\int_0^{\infty} e^{-x}\,dx$ converges, the series $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges.	Evaluation	**1 point**

(b)

$\lim_{n\to\infty} \frac{\frac{1}{e^n}}{\left\|\frac{(-1)^n}{2e^n + 3}\right\|} = \lim_{n\to\infty} \frac{2e^n + 3}{e^n} = 2$ The limit exists and is positive. Therefore, because the series $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges, the series $\sum_{n=0}^{\infty} \left\|\frac{(-1)^n}{2e^n + 3}\right\|$ converges by the limit comparison test. Thus, the series $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ converges absolutely.	Sets up limit comparison	**1 point**
	Explanation	**1 point**

(c)

$\left\lvert \dfrac{(-1)^{n+1} x^{n+1}}{2e^{n+1}+3} \cdot \dfrac{2e^n+3}{(-1)^n x^n} \right\rvert = \left\lvert \dfrac{(2e^n+3)x^{n+1}}{(2e^{n+1}+3)x^n} \right\rvert = \dfrac{2e^n+3}{2e^{n+1}+3}\lvert x\rvert$	Sets up ratio	**1 point**
$\lim\limits_{n\to\infty} \dfrac{2e^n+3}{2e^{n+1}+3}\lvert x\rvert = \dfrac{1}{e}\lvert x\rvert$	Computes limit of ratio	**1 point**
$\dfrac{1}{e}\lvert x\rvert < 1 \Rightarrow \lvert x\rvert < e$ The radius of convergence is $R = e$.	Answer	**1 point**

(d)

The terms of the alternating series $g(1) = \sum\limits_{n=0}^{\infty} \dfrac{(-1)^n}{2e^n+3}$ decrease in magnitude to 0. The alternating series error bound for the error of the approximation is the absolute value of the third term of the series. Error $\le \left\lvert \dfrac{(-1)^2}{2e^2+3} \right\rvert = \dfrac{1}{2e^2+3}$	Answer	**1 point**