AP Calculus Practice Exam
BC Version - Section I - Part A

Calculators ARE NOT Permitted On This Portion Of The Exam
28 Questions - 55 Minutes

$$3y^2 - 4e^{(-2x)} - xy = 4$$

1) Given
Find dy/dx.

a) $\frac{-4e^{(-2x)} - y}{6y - x}$ b) $-\frac{8e^{(-2x)}}{6y + x}$ c) $\frac{8e^{(-2x)}}{6y + x}$ d) $-\frac{8e^{(-2x)} - y}{6y - x}$

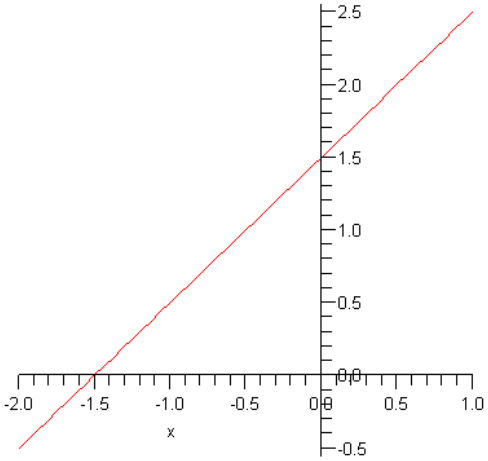
e) $\frac{8e^{(-2x)} - y}{6y - x}$

2) Give the volume of the solid generated by revolving the region bounded by the graph of $y = \ln(x)$, the x-axis, the lines $x = 1$ and $x = e$, about the y-axis.

a) $\frac{1}{2}\pi(e^4 - 1)$ b) $\frac{1}{4}\pi(e^2 + e)$ c) $\frac{1}{4}\pi(e^2 - e)$ d) $\frac{1}{2}\pi(e^4 + 1)$

e) $\frac{1}{2}\pi(e^2 + 1)$

3) The graph of the **derivative** of f is shown below.



Find the area bounded between the graph of f and the x-axis over the interval [-2,1], given that $f(0) = 1$.
a) 13/4 b) 29/12 c) 8/3 d) 31/12 e) 11/4

4) Determine dy/dt, given that $y = x^2 + 4x$ and $x = \cos(3t)$

a) $-6\cos(3t)\sin(3t)$ b) $-3(2\cos(3t)+4)\sin(3t)$

c) $6\cos(3t)+12$ d) $-18\sin(3t)$

e) $3(2\cos(3t)+4)\cos(3t)$

5) The function $f(x) = 5x^2 + 3e^{(2x)}$ is invertible. Give the slope of the normal line to the graph of f^{-1} at $x = 3$.

a) $-\frac{1}{30+6e^6}$ b) -2/3c) 1/6d) 30 + e^6e) -6

6) Determine $\int (\sin(6x))^2 (\cos(6x))^2 \, dx$

a) $\frac{1}{8}x - \frac{1}{192}\sin(24x) + C$

b) $\frac{1}{8}x + \frac{1}{96}\sin(12x) + C$

c) $\frac{1}{8}x - \frac{1}{192}\cos(24x) + C$

d) $\frac{1}{8}x + \frac{1}{192}\sin(24x) + C$

e) $\frac{1}{8}x - \frac{1}{96}\sin(12x) + C$

7) Give the polar representation for the circle of radius 2 centered at (0 , 2).

a) r = 2 sin(θ) + 2 cos(θ)

b) r = 4 cos(θ)

c) r = 4 sin(θ)

d) r sin(θ) = 2

e) r = 4 sin(θ) - cos(θ)

8) Determine

$$\lim_{t \to \infty} \left(4\, t^2 \left(\sin\left(\frac{2}{t}\right)\right)^2\right)$$

a) 2
b) 1
c) 16
d) ½
e) 32

9) Determine

$$\int_1^2 \frac{1}{\sqrt{4 - x^2}}\, dx$$

a) 2/3 π
b) 1/3 π
c) -1/3 π
d) 1/6 π
e) -1/6 π

10) Give the radius of convergence for the series

$$\sum_{k=1}^{\infty} \frac{3^{(k+2)}\, x^k}{k+1}$$

a) The series diverges for all X
b) 1
c) 0
d) 1/3
e) 3

11) Determine

$$\lim_{t \to \infty} \left(t \left(\ln\left(2 + \frac{1}{t}\right) - \ln(2)\right)\right)$$

a) $\frac{1}{2}\ln(2)$
b) 2
c) ½
d) 0
e) ∞

12) The position of a particle moving along the *x*-axis at time *t* is given by

$$x(t) = (\sin(4\pi t))^2$$

At which of the following values of *t* will the particle change direction?
I) $t = 1/8$
II) $t = 1/6$
III) $t = 1$
IV) $t = 2$

a) I, II and III
b) I and II
c) I, III and IV
d) II, III and IV
e) III and IV

13) Determine

$$\int_0^{\pi} x\cos(x)\,dx$$

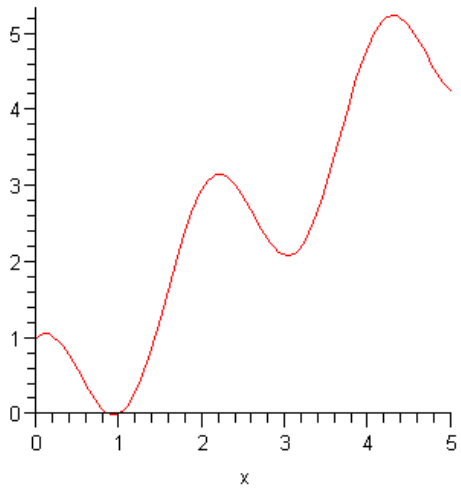
a) $-1 + \frac{1}{2}\pi$
b) π
c) -2
d) $-\frac{4}{3} + \frac{1}{3}\pi$
e) $-\frac{3}{2} + \frac{1}{4}\pi$

14) Determine the *y*-intercept of the tangent line to the curve

$$y = \sqrt{x^2 + 33}$$

at $x = 4$.
a) 45/7
b) 66/49
c) -33/49
d) 135/49
e) 33/7

15) The function *f* is graphed below.



Give the number of values of *c* that satisfy the conclusion of the Mean Value Theorem for derivatives on the interval [2,5].

a) 3
b) 2
c) 1
d) 4
e) 3.2

16) Give the average value of the function

$$f(x) = 2\,e^{(x-4)}$$

on the interval [1,3].

a) $e^{(-1)} + e^{(-2)}$

b) $-\frac{2}{3}\,e^{(-3)} + \frac{2}{3}\,e^{(-1)}$

c) $\frac{2}{3}\,e^{(-1)}$

d) $-e^{(-3)} + e^{(-1)}$

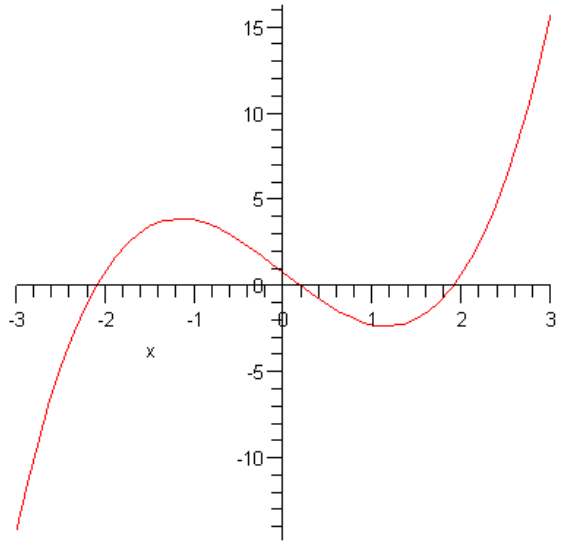
e) $-2\,e^{(-3)} + 2\,e^{(-1)}$

17) A rectangle has both a changing height and a changing width, but the height and width change so that the area of the rectangle is always 200 square feet. Give the rate of change of the width (in ft/sec) when the height is 5 feet, if the height is decreasing at that moment at the rate of 1/2 ft/sec.

a) 2/5
b) -2/5
c) 1/40

d) 1/60
e) 205

18) The graph of the **derivative** of *f* is shown below.



Give the number of values of *x* in the interval [-3,3] where the graph of *f* has inflection.
a) 1
b) 2
c) 0
d) 3
e) There is not enough information

19) A rectangle has its base on the *x*-axis and its vertices on the positive portion of the parabola

$$y = 2 - 3x^2$$

What is the maximum possible area of this rectangle?

a) $\frac{8}{27}\sqrt{3}\sqrt{6}$

b) $\frac{2}{9}\sqrt{3}\sqrt{6}$

c) $\frac{4}{15}\sqrt{5}\sqrt{6}$

d) $\frac{2}{15}\sqrt{5}\sqrt{6}$

e) $\frac{1}{3}\sqrt{2}\sqrt{6}$

20) Compute

$$\int e^{(2x)} \left(\tan\left(e^{(2x)}\right)\right)^2 dx$$

a) $\frac{1}{2} \left(\sec\left(e^{(2x)}\right)\right)^2 + C$

b) $\frac{1}{2} \tan\left(e^{(2x)}\right) e^{(2x)} + C$

c) $4 \tan\left(e^{(2x)}\right) \left(\sec\left(e^{(2x)}\right)\right)^2 e^{(2x)} + C$

d) $\frac{1}{2} \tan\left(e^{(2x)}\right) - \frac{1}{2} e^{(2x)} + C$

e) $\frac{1}{2} \tan\left(e^{(2x)}\right) + \frac{1}{2} e^{(2x)} + C$

21) Determine

$$\int_0^{\infty} \frac{1}{36 + x^2} dx$$

a) ½ π
b) 1/12 π
c) 3π
d) ∞
e) 6π

22) Determine

$$\lim_{x \to \infty} \left(4^x + 7^x\right)^{\left(\frac{1}{x}\right)}$$

a) 11/2
b) 7
c) ∞
d) e^7
e) 11

23) Give the exact value of

$$\sum_{n=0}^{\infty} \frac{\cos(n\pi)\, 5^n}{n!}$$

a) e^{-5}
b) sin (5)
c) cos (5)
d) e^{5}
e) – sin (5)

24) Determine

$$\lim_{x \to 0} \left(\frac{e^{x} + e^{(-x)} - 2}{1 - \cos(x)} \right)$$

a) 0
b) 1
c) 2
d) undefined
e) 3/2

25) Give the derivative of

$$f(x) = x^{(-2x)}$$

a) $-2x\, x^{(-2x)} - 2\, x^{(-2x)} \ln(x)$

b) $-2x\, x^{(-2x-1)} + x^{(-2x)} \ln(x)$

c) $x\, x^{(-2x-1)} - 2\, x^{(-2x)} \ln(x)$

d) $-2x\, x^{(-2x-1)} - 2\, x^{(-2x)} \ln(x)$

e) $-2x\, x^{(-2x-1)}$

26) Give the first 3 nonzero terms in the Taylor series expansion about $x = 0$ for the function

$$f(x) = \cos(2x)$$

a) $1 - 2x^2 + 4x^4$

$1 - 2x^2 + \frac{2}{3}x^4$

b)

$x - \frac{4}{3}x^3 + \frac{4}{15}x^5$

c)

$1 + 2x + 2x^2$

d)

$1 - 2x^2$

e)

27) Determine

$$\int \frac{x}{x^2 + 2x - 8}\,dx$$

$\frac{1}{2}\ln((x-2)\ (x+4)) + C$

a)

$\frac{1}{3}\ln(|x-2|) + \frac{2}{3}\ln(|x+4|) + C$

b)

$-\frac{2}{3}\ln(|x-2|) - \frac{1}{3}\ln(|x+4|) + C$

c)

$-\frac{2}{3}\ln(|x+4|) - \frac{1}{3}\ln(|x-2|) + C$

d)

$\frac{1}{2}\ln(|(x-2)\ (x+4)|) + C$

e)

28) Which of the following series converge(s)?

$$\left[A = \sum_{n=1}^{\infty} \frac{1}{n^{(3/4)}},\ B = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^5}},\ C = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}\right]$$

a) B only
b) A, B and C
c) B and C
d) A and B
e) A and C

AP Calculus Practice Exam
BC Version - Section I - Part B

Calculators ARE Permitted On This Portion Of The Exam
17 Questions - 50 Minutes

1) The limit of the sequence

$$u_n = \frac{1 + c\,n^2}{(2\,n + 3 + 2\sin(n))^2}$$

as *n* approaches ∞ is -3. What is the value of *c*?
a) -3/4
b) -9
c) -3/2
d) -12
e) -27

2) If

$$\frac{dy}{dx} = 4\,y\,x^2$$

and $y = 3$ when $x = -2$, then what is *y*?

a) $\dfrac{3\,e^{\left(-\frac{4}{3}x^3\right)}}{e^{\left(\frac{32}{3}\right)}}$

b) $\dfrac{3\,e^{\left(\frac{4}{3}x^2\right)}}{e^{\left(\frac{16}{3}\right)}}$

c) $\dfrac{3\,e^{\left(\frac{4}{3}x\right)}}{e^{\left(\frac{-8}{3}\right)}}$

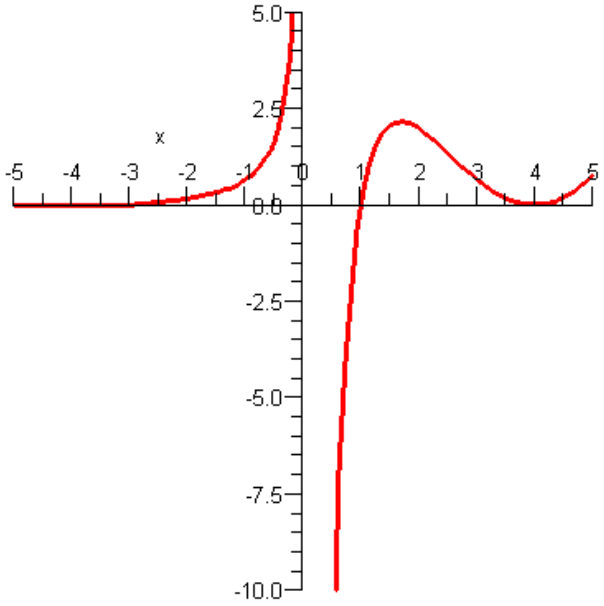
$$\frac{3\,e^{\left(-\frac{4}{3}x^2\right)}}{e^{\left(\frac{16}{3}\right)}}$$

d)

$$\frac{3\,e^{\left(\frac{4}{3}x^3\right)}}{e^{\left(\frac{-32}{3}\right)}}$$

e)

3) The graph of the **derivative** of *f* is given below.



Which of the following is FALSE about the function *f*?

a) *f* is increasing on [1,4].
b) *f* is concave down on [1,5/2].
c) *f* is concave down on [-3,0).
d) *f* is not differentiable at 0.
e) The funciton is constant on (- ∞ ,-3].

4) Determine

$$\int_{-8}^{1} \frac{1}{x^{(5/3)}}\,dx$$

a) -3/2
b) The integral does not exist
c) 3/2

d) 3/4
e) 1/12

5) Give the area that lies below the *x*-axis and is contained within the region bounded by the polar curve

$$r = 1 + 2\sin(\theta)$$

a) $\pi - 4 + \frac{1}{2}\sqrt{3}$

b) $\frac{1}{4}\pi - 2 + \frac{1}{4}\sqrt{3}$

c) $\frac{1}{2}\pi - 2 + \frac{3}{2}\sqrt{3}$

d) $\pi - 2 + \frac{3}{2}\sqrt{3}$

e) $\frac{1}{2}\pi - 4 + \frac{3}{2}\sqrt{3}$

6) Give the error that occurs when the area between the curve

$$y = x^3 + 1$$

and the *x*-axis over the interval [0,1] is approximated by the trapezoid rule with $n = 4$.
a) 0.016
b) 0.046
c) 0.025
d) 0.128
e) 0.008

7) Let

$$f(x) = \sum_{k=0}^{\infty} (1 - (\sin(x))^2)^k$$

Determine $f(2\pi/3)$.
a) ∞
b) 2
c) 4/3
d) ½
e) 3/2

8) Give the length of the curve determined by

$$[x = 4t^2,\ y = t^3 + 2t]$$

for *t* from 0 to 2.
a) 20.0849
b) 20.1390
c) 20.1084
d) 20.0735
e) 20.0886

9) Particles A and B leave the origin at the same time and move along the *y*-axis. Their positions are determined by the functions

$$[\, y_A = 2\sin(2t),\ y_B = 4\cos(t)\,]$$

for *t* between 0 and 8. What is the velocity of particle B when particle A stops for the first time?
a) 0
b) $-2\sqrt{2}$
c) -4
d) 4
e) $2\sqrt{2}$

10) The base of a solid is the region in the *xy* plane enclosed by the curves

$$[\, f(x) = \sin(x),\ g(x) = \cos(x)\,]$$

over the interval [0, π/4]. Cross sections of the solid perpendicular to the *x*-axis are squares. Determine the volume of the solid.
a) 0.3061
b) 0.2564
c) 0.3146
d) 0.2855
e) 0.2572

11) Give the minimum value of the function

$$f(x) = 2x^3 - 9x + 5$$

for $x \geq 0$.
a) -2.258
b) -2.368
c) -2.349
d) -2.213
e) -2.175

12) Select the TRUE statement associated with the function

$$f(x) = \frac{\sin(x)}{x^2}$$

a) The graph of the function passes through the origin.
b) The function does not have a horizontal asymptote.
c) The function has a vertical asymptote at $x = 0$.
d) The graph of the function is symmetric about the x-axis.
e) The graph is always concave up.

13) The function g is the derivative of

$$\int_0^x (t^3 - 5)\, dt$$

What is the derivative of the inverse of g at $x = 3$?
a) 12
b) ¼
c) 1/12
d) 4
e) 1/3

14) The half-life of radium-226 is 1625 years. What percentage of a given amount of the radium will remain after 1000 years?
a) 65.34%
b) 65.20%
c) 65.25%
d) 65.35%
e) 65.30%

15) The function f satisfies the equation

$$\int_0^{2x} f(t)\, dt = 4 \sin(x) + x$$

Evaluate $f(\pi/3)$.
a) 2.232
b) 1.500
c) 4.464
d) 3
e) 2.432

16) A rectangular box with square base and top is to be made to contain 2160 cubic feet. The material for the base costs 30 cents per square foot, the material for the top costs 50 cents per square foot, and the material for the sides costs 20 cents per square foot. Give the length of one side of the base (in feet) so that the cost is minimized.
a) 9.16
b) 9.78
c) 8.32
d) 10.26
e) 12.38

17) Which expression represents the volume of the solid generated when the region between the curves

$$\left[y = 6 - x^2,\ y = \frac{1}{2} x^2 \right]$$

over the interval [0,2] is rotated around the *x*-axis?

a) $2\pi \int_2^6 y\sqrt{6-y}\ dy + 2\pi \int_0^2 y^{(3/2)}\sqrt{2}\ dy$

b) $2\pi \int_0^2 y\sqrt{6-y}\ dy - 2\pi \int_0^2 y^{(3/2)}\sqrt{2}\ dy$

c) $2\pi \int_2^6 y\sqrt{6-y}\ dy - 2\pi \int_0^2 y^{(3/2)}\sqrt{2}\ dy$

d) $2\pi \int_0^6 y\sqrt{6-y}\ dy + 2\pi \int_0^2 y^{(3/2)}\sqrt{2}\ dy$

e) $2\pi \int_0^6 y\sqrt{6-y}\ dy - 2\pi \int_0^2 y^{(3/2)}\sqrt{2}\ dy$

Part A

1) d)
2) e)
3) b)
4) b)
5) e)
6) a)
7) c)
8) c)
9) b)
10) d)
11) c)
12) c)
13) c)
14) e)
15) a)
16) d)
17) a)
18) b)
19) a)
20) d)
21) b)
22) b)
23) a)
24) c)
25) d)
26) b)
27) b)
28) c)

Part B

1) d)
2) e)
3) b)
4) b)
5) e)
6) a)
7) c)
8) c)
9) b)
10) d)
11) c)
12) c)
13) c)
14) e)
15) a)
16) d)
17) a)